Published by Institute of Physics Publishing for SISSA

Received: *April 20, 2006*
Revised: *May 25, 2006*
Accepted: *June 2, 2006*
Published: *June 26, 2006*

Dynamical aspects of the plane-wave matrix model at finite temperature

Naoyuki Kawahara,[ab] **Jun Nishimura**[ac] **and Kentaroh Yoshida**[a]

[a] *High Energy Accelerator Research Organization (KEK)*
1-1 Oho, Tsukuba 305-0801, Japan
[b] *Department of Physics, Kyushu University*
Fukuoka 812-8581, Japan
[c] *Department of Particle and Nuclear Physics*
Graduate University for Advanced Studies (SOKENDAI)
1-1 Oho, Tsukuba 305-0801, Japan
E-mail: kawahara@post.kek.jp, jnishi@post.kek.jp, kyoshida@post.kek.jp

Abstract: We study dynamical aspects of the plane-wave matrix model at finite temperature. One-loop calculation around general classical vacua is performed using the background field method, and the integration over the gauge field moduli is carried out both analytically and numerically. In addition to the trivial vacuum, which corresponds to a single M5-brane at zero temperature, we consider general static fuzzy-sphere type configurations. They are all 1/2 BPS, and hence degenerate at zero temperature due to supersymmetry. This degeneracy is resolved, however, at finite temperature, and we identify the configuration that gives the smallest free energy at each temperature. The Hagedorn transition in each vacuum is studied by using the eigenvalue density method for the gauge field moduli, and the free energy as well as the Polyakov line is obtained analytically near the critical point. This reveals the existence of fuzzy sphere phases, which may correspond to the plasma-ball phases in $\mathcal{N} = 4$ SU(∞) SYM on $S^1 \times S^3$. We also perform Monte Carlo simulation to integrate over the gauge field moduli. While this confirms the validity of the analytic results near the critical point, it also shows that the trivial vacuum gives the smallest free energy throughout the high temperature regime.

Keywords: Penrose limit and pp-wave background, M-Theory, Thermal Field Theory, M(atrix) Theories.

Contents

1. Introduction

Matrix models provide a promising approach to non-perturbative dynamics of superstring theories and M-theory [1 − 3]. In particular the matrix model proposed by Banks, Fischler, Shenker and Susskind (BFSS) [1] is conjectured to be a non-perturbative formulation of M-theory, a hypothetical theory in eleven dimensions, whose low energy effective theory is given by 11d supergravity. The BFSS matrix model, which takes the form of a matrix quantum mechanics, can be obtained also from the supermembrane theory in eleven dimensions via the matrix regularization [4]. Studying the BFSS matrix model at finite temperature is an interesting subject due to its relation to the Schwarzschild black hole [5 − 7]. Such studies have been performed, for instance, in refs. [8, 9]. Applying matrix models to cosmology is another fascinating direction, which has developed recently [10].

While the BFSS matrix model is defined on a flat Minkowski space-time, we may consider matrix models also on curved space-time. In this regard the so-called pp-wave background provides a particularly simple and tractable example. In fact the maximally supersymmetric pp-wave background in eleven dimensions, which preserves 32 supercharges, is unique and its explicit form is known [11]. Berenstein, Maldacena and Nastase [12] proposed a matrix model on this background, which is closely related to the supermembrane theory on the pp-wave through the matrix regularization [13 – 15]. It has fuzzy spheres as classical solutions due to the presence of the mass term and the Myers term [16]. The stability of these solutions [13, 17, 18], as well as the spectrum around them [13, 19 – 21], has been studied intensively, while more general classical solutions such as a rotating fuzzy sphere are also discussed [22 – 25, 18, 26].[1] In fact the fuzzy sphere solutions can be interpreted as giant gravitons, and the interaction potential between them[2] is shown to be of the $1/r^7$-type [18, 26] similarly to the results in the BFSS matrix model. This type of potential is anticipated since the spectrum in the linearized eleven-dimensional supergravity around the pp-wave background [32] is included in the matrix model as the spectrum of the zero-mode Hamiltonian [20, 15].

The plane-wave matrix model has been studied also at finite temperature by various authors. In ref. [33] the free energy around the trivial vacuum, which corresponds to a transverse M5-brane [34] at zero temperature,[3] was evaluated at the one-loop level, and the Hagedorn transition was studied in detail. (See refs. [36, 37] for a two-loop extension and ref. [38] for a review on this subject.)

In this paper we extend such calculations to general classical vacua, and identify the vacuum that gives the smallest free energy at each temperature. Similar studies have been made earlier in refs. [39, 40]. However, the degrees of freedom corresponding to the gauge field moduli, which appear at finite temperature, were not taken into account although they actually play an important role in the Hagedorn transition [33]. Here we perform the one-loop calculation by the background field method keeping the gauge field moduli arbitrary. The integration over the gauge field moduli is performed both analytically and numerically. Near the critical point we use the eigenvalue density method to obtain analytical results extending the previous works [33, 38] for the trivial vacuum. This reveals the existence of fuzzy-sphere phases, which may correspond to the plasma-ball phases in $\mathcal{N} = 4$ SU(∞) SYM on $S^1 \times S^3$ [41]. We also perform Monte Carlo simulation to integrate over the gauge field moduli with the matrix size up to $N = 2000$, which allows a reliable extrapolation to $N = \infty$. (Note that the results of ref. [39, 40] are obtained for small N.) While this confirms the validity of our analytical results, it also shows that the trivial vacuum gives the smallest free energy throughout the high temperature regime. In fact the leading asymptotic behavior in the high temperature limit is universal, but the difference appears in the subleading term. Our conclusion is different from the previous works [39, 40]. This is due to the Hagedorn transition, which occurs only if we include the gauge field moduli and take the large N limit. As a result, the behavior of the free energy changes drastically.

[1] A matrix string theory on a type IIA plane-wave background, which includes fuzzy spheres as classical solutions, has originally been constructed in [27]. The spectrum around the fuzzy spheres was computed in [28] by following the method of [13]. The theory is applicable to the matrix cosmology scenario [29].

[2] The interaction potential between point-like gravitons has been studied recently in ref. [30]. See also ref. [31] for a short review on the graviton potential.

[3] In a deformed plane-wave matrix model with an interaction term coming from the 6-form potential, a fuzzy five-sphere solution was constructed [35].

JHEP06(2006)052

Comparison of the free energy for general classical vacua was also made in a matrix model, which is obtained by dimensionally reducing the plane-wave matrix model to a point [42]. There the true vacuum turned out to be a complicated multi-fuzzy-sphere type configuration, which incorporates a non-trivial gauge group.[4] Analogous models without the mass term have been studied intensively by Monte Carlo simulation [43]. These studies are partly motivated from the IIB matrix model [2], a conjectured nonperturbative formulation of type IIB superstring theory, which can be formally obtained by dimensionally reducing the BFSS matrix model to a point. More intimate relationship between the two models has been suggested [44] on account of the large N reduction [45]. Calculation of the free energy in the IIB matrix model for various configurations provided certain evidences that four-dimensional space-time is favored dynamically [46]. See ref. [47] for related works on this issue.

The results obtained in the present paper are expected to have close relationship to the phase structure of large N gauge theories at finite temperature and at finite volume, which has been explored in refs. [48 − 50, 41, 51]. According to the AdS/CFT correspondence [52] at finite temperature [53], the observed phase transitions should correspond to those on the gravity side such as the Hawking-Page phase transition [54] and the Gregory-Laflamme phase transition [55]. This provides an explicit realization of the connection between the Hagedorn transition in string theory and the deconfinement phase transition in large N gauge theories pointed out earlier [56].

The rest of this paper is organized as follows. In section 2 we briefly review the plane-wave matrix model and its classical vacua. In section 3 we perform one-loop calculation around general classical vacua keeping the gauge field moduli arbitrary. In section 4 we perform the integration over the gauge field moduli to obtain explicit results for the free energy and the Polyakov line. Section 5 is devoted to a summary and discussions. In the appendices we present the details of our calculations.

2. The plane-wave matrix model and its classical solutions

In this section we define the plane-wave matrix model and discuss its classical solutions. In order to study the model at finite temperature, we make the Wick rotation $t \to -it$ and compactify the imaginary time direction to a circle with the circumference $\beta \equiv 1/T$. Thus the action can be written as[5]

$$S_{\rm pp} = \int_0^\beta dt \, \mathrm{Tr} \left[\frac{1}{2}(D_t X_M)^2 - \frac{1}{4}[X_M, X_N]^2 + \frac{1}{2}\Psi^\dagger D_t \Psi - \frac{1}{2}\Psi^\dagger \gamma_M [X_M, \Psi] \right.$$
$$\left. + \frac{1}{2}\left(\frac{\mu}{3}\right)^2 (X_i)^2 + \frac{1}{2}\left(\frac{\mu}{6}\right)^2 (X_a)^2 + i\frac{\mu}{3}\epsilon_{ijk} X_i X_j X_k + i\frac{\mu}{8}\Psi^\dagger \gamma_{123}\Psi \right], \qquad (2.1)$$

[4]Note that the model studied there is closely related but not equivalent to the high temperature limit of the plane-wave matrix model.

[5]We have made the rescaling, $A \to RA$, $t \to \frac{1}{R}t$, $\mu \to R\mu$, in the action presented, for instance, in ref. [14] to arrive at the present form, which does not include the parameter R (the compactification radius of the 11th direction in M-theory) explicitly.

where $D_t = \partial_t - i[A, \ \cdot \ ]$ represents the covariant derivative, and the indices M, N run from 1 to 9. Since the transverse SO(9) symmetry is broken down to SO(3) $\times$ SO(6) on the pp-wave background, we have also introduced the SO(3) indices $i, j, k = 1, 2, 3$ and the SO(6) indices $a = 4, \ldots, 9$. The partition function for the finite temperature system is defined by

$$Z = \int [dA(t)][dX_M(t)][d\Psi(t)][d\Psi^\dagger(t)] \exp\left(-S_{\rm pp}\right) , \tag{2.2}$$

where the bosonic and fermionic fields obey the periodic and anti-periodic boundary conditions, respectively, in the imaginary time direction.

Let us discuss static classical solutions of the model within the usual Ansatz $X_a = \Psi = 0$ [12]. At zero temperature it is convenient to set the one-dimensional gauge field $A(t)$ to zero by choosing the gauge, while at finite temperature we can only set $A(t)$ to a constant diagonal matrix due to the nontrivial holonomy in the temporal direction. In the case of the trivial vacuum $X_i = 0$ $(i = 1, 2, 3)$, for instance, the gauge field can be an arbitrary constant diagonal matrix without increasing the action, and the corresponding moduli parameters should be integrated. These degrees of freedom, which we refer to as the gauge field moduli, indeed play a crucial role in the Hagedorn transition [33].

In fact the plane-wave matrix model has static fuzzy-sphere type classical solutions [12]

$$X_i = B_i \equiv \frac{\mu}{3} L_i = \frac{\mu}{3} \bigoplus_{I=1}^{s} \left(L_i^{(n_I)} \otimes \mathbf{1}_{k_I} \right) , \tag{2.3}$$

$$A(t) = \bar{A}(t) \equiv \bigoplus_{I=1}^{s} \left(\mathbf{1}_{n_I} \otimes \bar{A}^{(I)}(t) \right) , \tag{2.4}$$

where $L_i^{(n)}$ are the SU(2) generators in the n-dimensional irreducible representation satisfying $[L_i^{(n)}, L_j^{(n)}] = i\epsilon_{ijk} L_k^{(n)}$, and the parameters k_I and n_I obey $\sum_{I=1}^{s} n_I \cdot k_I = N$. The $k_I \times k_I$ matrices $\bar{A}^{(I)}(t)$ represent the gauge field moduli. One can easily check that the classical action for the above solution is zero for arbitrary $\bar{A}^{(I)}(t)$ by noting that $[B^i, \bar{A}(t)] = 0$.

At zero temperature, these solutions are interpreted as stacks of M2-branes or M5-branes (or their mixture) depending on how one takes the large N limit [34]. In what follows we often restrict ourselves to the $s = 1$ case for simplicity, and use the parameters $n \equiv n_1$ and $k \equiv k_1$, which satisfy $n \cdot k = N$. Then the solution represents k copies of spherical M2-branes if k is held fixed, while it represents n copies of spherical M5-branes if n is held fixed, in the large N limit. An evidence for the latter interpretation is provided by the coincidence of the spectrum [34] for the trivial vacuum $(n = 1)$ case. Note, however, that the argument for identifying transverse M5-branes in the plane-wave matrix model is based on the protected BPS multiplet, and it relies rather crucially on supersymmetry, which is broken at finite temperature. Hence it is not totally clear whether the M5-branes are stable against thermal fluctuations. One way to demonstrate the existence of M5-branes is to compute their radius at finite temperature by the Gaussian expansion method extending the calculation in ref. [7] to the present case.

JHEP06(2006)052

3. One-loop effective action

In this section we compute the one-loop effective action around the classical solutions keeping the gauge field moduli arbitrary using the background field method. The matrices are decomposed as

$$
\begin{aligned}
X_i(t) &= B_i + Y_i(t)\,, \qquad & X_a(t) &= 0 + Y_a(t)\,, \\
A(t) &= \bar{A}(t) + \tilde{A}(t)\,, \qquad & \Psi(t) &= 0 + \Psi(t)\,,
\end{aligned} \tag{3.1}
$$

where $Y_i(t)$, $Y_a(t)$, $\tilde{A}(t)$ and $\Psi(t)$ represent the fluctuations around the background B_i and $\bar{A}(t)$.

For the trivial vacuum $X_i = 0$, since the fluctuation $\tilde{A}(t)$ can be totally absorbed into the gauge field moduli $\bar{A}(t)$, we set $\tilde{A}(t)$ to zero. Similarly, in the general case, we omit $\sum_{I=1}^{s}(k_I)^2$ zero modes in the fluctuation $\tilde{A}(t)$, which corresponds to changing the gauge field moduli. In the presence of the background, the $\mathrm{U}(N)$ gauge symmetry is broken down to $\prod_{I=1}^{s}\mathrm{U}(k_I)$. Therefore, we also obtain zero modes in the direction of the gauge orbit corresponding to the broken symmetry. These zero modes should be removed by an appropriate gauge fixing, and we use the background field gauge for this purpose. By using the unbroken gauge symmetry, the gauge field moduli $\bar{A}^{(I)}(t)$ can be brought into a static diagonal form

$$
\bar{A}^{(I)}(t) = \frac{1}{\beta}\,\mathrm{diag}(\alpha_1^{(I)}, \ldots, \alpha_{k_I}^{(I)})\,, \tag{3.2}
$$

where $\alpha_a \in (-\pi, \pi]$. See appendix A for the details of the two gauge fixing procedures.

In the rest of this section, we integrate out the fluctuations perturbatively keeping the moduli parameters $\alpha_a^{(I)}$ $(a = 1, \ldots, k_I)$ arbitrary. We add the gauge fixing term $S_{\text{g.f.}}$ and the ghost term S_{ghost} given by eq. (A.6) to the action, and expand it with respect to the fluctuations as $S = S^{(0)} + S^{(1)} + \cdots + S^{(4)}$. The classical action $S^{(0)}$ vanishes for the backgrounds we consider here, and the linear term $S^{(1)}$ vanishes, too, since the backgrounds satisfy the classical equations of motion. The quadratic term is given by

$$
\begin{aligned}
S^{(2)} = \int_0^\beta dt\,\mathrm{Tr}\Bigg[&\frac{1}{2}(\overline{D}_t Y_i)^2 - [Y_i, Y_j][B_i, B_j] - \frac{1}{2}[Y_i, B_j]^2 + \frac{1}{2}\left(\frac{\mu}{3}\right)^2 Y_i^2 \\
&-i\frac{\mu}{2}\epsilon_{ijk} Y_i\,[B_k, Y_j] + \frac{1}{2}(\overline{D}_t Y_a)^2 - \frac{1}{2}[Y_a, B_i]^2 + \frac{1}{2}\left(\frac{\mu}{6}\right)^2 Y_a^2 \\
&+\psi^{\dagger A\alpha}\overline{D}_t\psi_{A\alpha} + \psi^{\dagger A\alpha}\sigma_{i\alpha}^{\beta}[B_i, \psi_{A\beta}] + \frac{\mu}{4}\psi^{\dagger A\alpha}\psi_{A\alpha} \\
&+\frac{1}{2}(\overline{D}_t\tilde{A})^2 - \frac{1}{2}[\tilde{A}, B_i]^2 + \overline{D}_t\bar{c}\cdot\overline{D}_t c - [B_i, \bar{c}][B_i, c]\Bigg]\,,
\end{aligned} \tag{3.3}
$$

where c and $\bar{c}$ are the Faddeev-Popov ghosts. We have decomposed the spinor indices according to $\mathrm{SU}(2)\times\mathrm{SU}(4)$, where $\alpha = 1, 2$ and $A = 1, \ldots, 4$ represent $\mathrm{SU}(2)$ and $\mathrm{SU}(4)$ indices, respectively [13]. The contributions of the higher order terms can be neglected in the $\mu \to \infty$ limit, as can be seen by rescaling of the variables as

$$
Y_M \to \mu^{-1/2} Y_M\,, \quad \tilde{A} \to \mu^{-1/2}\tilde{A}\,, \quad \bar{A} \to \mu\bar{A}\,,
$$

$$c \to \mu^{-1/2} c\,, \quad \bar{c} \to \mu^{-1/2}\bar{c}\,, \quad t \to \mu^{-1} t\,, \tag{3.4}$$

which brings the action S into the form

$$S = S^{(2)} + \mu^{-3/2} S^{(3)} + \mu^{-3} S^{(4)}\,,$$

where μ that appear in $S^{(2)}$, $S^{(3)}$ and $S^{(4)}$ are set to unity. Hence the one-loop calculation is justified in the large μ limit. Since we also take the large N limit, it is important to clarify the N dependence of the expansion parameter. Let us recall that the expansion parameter in perturbation theory [13] is given by $\frac{NR^3}{\mu^3} = \frac{N^4}{(\mu p^+)^3}$, where we have temporarily restored the parameter R. Clearly the expansion parameter diverges in the large N limit with fixed μ and $p^+ = N/R$. Hence, in order to ensure the validity of the perturbative expansion, we need to take the $\mu \to \infty$ limit faster than the large N limit. Note also that non-perturbative effects such as the tunneling between the trivial vacuum and the fuzzy sphere vacua is suppressed due to the $N \to \infty$ and $\mu \to \infty$ limits [57].

Within the perturbation theory we are considering, it is natural to consider the temperature in units of μ. This implies that the temperature should be sent to infinity in the $\mu \to \infty$ limit. Note, however, that our analysis is valid for finite but large N and μ as far as μ is taken to be sufficiently large for a given value of N. Therefore, we do not need to take the strict $T \to \infty$ limit, and we do obtain explicit results for arbitrary finite temperature. With this understanding, we set $\mu = 1$ to simplify the expressions in what follows, but one can restore the μ dependence of the results after integrating the fluctuations by simply replacing β by $\beta\mu$ (i.e., replacing T by $\frac{T}{\mu}$).

Let us introduce the operators

$$\mathcal{L}_i M \equiv [L_i, M]\,, \tag{3.5}$$

which act on an $N \times N$ matrix M. Then the quadratic term $S^{(2)}$ in the action can be rewritten as

$$\begin{aligned} S^{(2)} = \int_0^\beta dt\, \mathrm{Tr}\Big[&\frac{1}{2}(\overline{D}_t Y_i)^2 + \frac{1}{2} Y_i P_{ij}(\mathcal{L}) Y_j + \frac{1}{2}(\overline{D}_t Y_a)^2 + \frac{1}{2} Y_a Q(\mathcal{L}) Y_a \\ &+ \psi^{\dagger A\alpha}\overline{D}_t \psi_{A\alpha} + \psi^{\dagger A\alpha} R_\alpha^\beta(\mathcal{L})\psi_{A\alpha} + \frac{1}{2}(\overline{D}_t \tilde{A})^2 + \frac{1}{2}\tilde{A} T(\mathcal{L})\tilde{A} + \overline{D}_t \bar{c}\overline{D}_t c + \bar{c} T(\mathcal{L}) c\Big]\,, \end{aligned} \tag{3.6}$$

where we have defined the following mass operators

$$\begin{aligned} P_{ij}(\mathcal{L}) &= \frac{1}{9}\left\{\left(\mathcal{L}_k{}^2 + 1\right)\delta_{ij} - i\epsilon_{ijk}\mathcal{L}_k\right\}\,, \qquad Q(\mathcal{L}) = \frac{1}{9}\left(\mathcal{L}_k{}^2 + \frac{1}{4}\right)\,, \\ R_\alpha^\beta(\mathcal{L}) &= \frac{1}{4}\delta_\alpha^\beta + \frac{1}{3}\sigma_{k\alpha}^\beta \mathcal{L}_k\,, \qquad T(\mathcal{L}) = \frac{1}{9}\mathcal{L}_k{}^2\,. \end{aligned} \tag{3.7}$$

The mass spectra, given by the eigenvalues of these operators, have been obtained for the physical modes [13] and for the unphysical modes [18]. Due to the structure (2.3) of L_i, it suffices to solve the eigenvalue problem for the $(n_I \times n_I)$ square block and the $(n_I \times n_J)$ rectangular block in the matrix, on which the mass operators act. The results are summarized in tables 1 and 2, respectively. Note that the mass spectra for the square

JHEP06(2006)052

type of fluctuations	mass	spins	degeneracy
Y_i (i=1,2,3)	$\frac{1}{3}\sqrt{l(l+1)}$	$1 \le l \le n_I - 1$	$2l+1$
	$\frac{1}{3}(l+1)$	$0 \le l \le n_I - 2$	$2l+1$
	$\frac{1}{3}l$	$1 \le l \le n_I$	$2l+1$
$Y_a(a=4,\ldots,9)$	$\frac{1}{3}(l+\frac{1}{2})$	$0 \le l \le n_I - 1$	$6(2l+1)$
ψ (fermion)	$(\frac{l}{3}+\frac{1}{4})$	$\frac{1}{2} \le l \le n_I - \frac{3}{2}$	$4(2l+1)$
	$(\frac{l}{3}+\frac{1}{12})$	$\frac{1}{2} \le l \le n_I - \frac{1}{2}$	$4(2l+1)$
$\tilde{A}$ (gauge)	$\frac{1}{3}\sqrt{l(l+1)}$	$1 \le l \le n_I - 1$	$2l+1$
$c, \bar{c}$ (ghost)	$\frac{1}{3}\sqrt{l(l+1)}$	$1 \le l \le n_I - 1$	$2l+1$

Table 1: Mass spectrum for the $(n_I \times n_I)$ square block.

type of fluctuations	mass	spins	degeneracy
$Y_i(i=1,2,3)$	$\frac{1}{3}\sqrt{l(l+1)}$	$\frac{1}{2}\lvert n_I - n_J\rvert \le l \le \frac{1}{2}(n_I+n_J)-1$	$2l+1$
	$\frac{1}{3}(l+1)$	$\frac{1}{2}\lvert n_I - n_J\rvert - 1 \le l \le \frac{1}{2}(n_I+n_J)-2$	$2l+1$
	$\frac{1}{3}l$	$\frac{1}{2}\lvert n_I - n_J\rvert + 1 \le l \le \frac{1}{2}(n_I+n_J)$	$2l+1$
$Y_a(a=4,\ldots,9)$	$\frac{1}{3}(l+\frac{1}{2})$	$\frac{1}{2}\lvert n_I - n_J\rvert \le l \le \frac{1}{2}(n_I+n_J)-1$	$6(2l+1)$
ψ (fermion)	$(\frac{l}{3}+\frac{1}{4})$	$\frac{1}{2}\lvert n_I - n_J\rvert - \frac{1}{2} \le l \le \frac{1}{2}(n_I+n_J)-\frac{3}{2}$	$4(2l+1)$
	$(\frac{l}{3}+\frac{1}{12})$	$\frac{1}{2}\lvert n_I - n_J\rvert + \frac{1}{2} \le l \le \frac{1}{2}(n_I+n_J)-\frac{1}{2}$	$4(2l+1)$
$\tilde{A}$ (gauge)	$\frac{1}{3}\sqrt{l(l+1)}$	$\frac{1}{2}\lvert n_I - n_J\rvert \le l \le \frac{1}{2}(n_I+n_J)-1$	$2l+1$
$c, \bar{c}$ (ghost)	$\frac{1}{3}\sqrt{l(l+1)}$	$\frac{1}{2}\lvert n_I - n_J\rvert \le l \le \frac{1}{2}(n_I+n_J)-1$	$2l+1$

Table 2: Mass spectrum for the $(n_I \times n_J)$ rectangular block.

block in the gauge and ghost fluctuations do not contain massless modes unlike the spectra obtained in ref. [18] for the vanishing background gauge field. In fact those massless modes are treated appropriately in the present formulation. The massless modes in the gauge field are nothing but the gauge field moduli. The massless modes in the ghost field are used for the gauge fixing, which brings the gauge field moduli into the diagonal form. The diagonal elements are integrated eventually. This way of treating the massless modes is specific to the finite temperature case.

Integrating out the fluctuations, we obtain the determinants of operators appearing in the quadratic action (3.6), which can be evaluated in a standard way, once the mass spectra are given. (See appendix B.) Including the Vandermonde determinant, which comes from the gauge fixing for the moduli integration (See appendix A.2.), the partition function is obtained at the one-loop level as

$$Z \equiv \mathcal{C} \int [d\alpha] \exp(-S_{\text{eff}}[\alpha]) \,, \tag{3.8}$$

where $S_{\text{eff}}[\alpha]$ represents the effective action for the moduli parameters $\alpha_a^{(I)}$, and the integration measure $[d\alpha]$ is given by

$$[d\alpha] = \prod_{I=1}^{s} \frac{1}{k_I!} \prod_{a=1}^{k_I} \frac{d\alpha_a^{(I)}}{4\pi} .$$

The normalization constant $\mathcal{C}$ turns out to be the same for all the classical solutions, as we demonstrate in the appendix C, and therefore it is irrelevant.

For the trivial vacuum, the masses are given by $\frac{1}{3}$, $\frac{1}{6}$ and $\frac{1}{4}$ for the fluctuations Y_i, Y_a and ψ, respectively, and the one-loop effective action $S_{\rm eff}$ reads [33]

$$\begin{aligned} S_{\rm eff} = &\sum_{a,b=1}^{N} \left[3 \ln \sinh \left\{ \frac{1}{2} \left(\frac{1}{3}\beta + i(\alpha_a - \alpha_b) \right) \right\} + 6 \ln \sinh \left\{ \frac{1}{2} \left(\frac{1}{6}\beta + i(\alpha_a - \alpha_b) \right) \right\} \right. \\ &\left. -8 \ln \cosh \left\{ \frac{1}{2} \left(\frac{1}{4}\beta + i(\alpha_a - \alpha_b) \right) \right\} \right] - \sum_{a \neq b}^{N} \left\{ \ln \left| \sin \left(\frac{1}{2}(\alpha_a - \alpha_b) \right) \right| \right\} . \end{aligned} \tag{3.9}$$

In the general case, the one-loop effective action $S_{\rm eff}$ can be decomposed as

$$S_{\rm eff} = \sum_{I=1}^{s} S_{\rm eff}^{(I)} + \sum_{I \neq J}^{s} S_{\rm eff}^{(I,J)} , \tag{3.10}$$

where the first (second) term comes from integrating the square (rectangular) block of the fluctuations, which corresponds to the interaction between the "fuzzy spheres" represented by matrices of the same (different) size, respectively. One series of the spectrum for Y_i gives $\frac{1}{3}\sqrt{l(l+1)}$, which coincides, including degeneracy, with the spectrum for the gauge field fluctuation $\tilde{A}$ and ghost fields c, $\bar{c}$. These are the unphysical modes, which cancel each other exactly. Thus, each term in eq. (3.10) is given by

$$\begin{aligned} S_{\rm eff}^{(I)} = &\sum_{a,b=1}^{k_I} \left[\sum_{l=0}^{n_I - 2} (2l+1) \ln \sinh \left\{ \frac{1}{2} \left(\frac{1}{3}\beta(l+1) + i \left(\alpha_a^{(I)} - \alpha_b^{(I)} \right) \right) \right\} \right. \\ &+ \sum_{l=1}^{n_I} (2l+1) \ln \sinh \left\{ \frac{1}{2} \left(\frac{1}{3}\beta l + i \left(\alpha_a^{(I)} - \alpha_b^{(I)} \right) \right) \right\} \\ &+ \sum_{l=0}^{n_I - 1} 6(2l+1) \ln \sinh \left\{ \frac{1}{2} \left(\frac{1}{3}\beta \left(l + \frac{1}{2} \right) + i \left(\alpha_a^{(I)} - \alpha_b^{(I)} \right) \right) \right\} \\ &- \sum_{l=\frac{1}{2}}^{n_I - \frac{3}{2}} 4(2l+1) \ln \cosh \left\{ \frac{1}{2} \left(\beta \left(\frac{l}{3} + \frac{1}{4} \right) + i \left(\alpha_a^{(I)} - \alpha_b^{(I)} \right) \right) \right\} \\ &\left. - \sum_{l=\frac{1}{2}}^{n_I - \frac{1}{2}} 4(2l+1) \ln \cosh \left\{ \frac{1}{2} \left(\beta \left(\frac{l}{3} + \frac{1}{12} \right) + i \left(\alpha_a^{(I)} - \alpha_b^{(I)} \right) \right) \right\} \right] \\ &- \sum_{a \neq b}^{k_I} \ln \left| \sin \frac{\alpha_a^{(I)} - \alpha_b^{(I)}}{2} \right| , \end{aligned} \tag{3.11}$$

$$
\begin{aligned}
S_{\text{eff}}^{(I,J)} = \sum_{a=1}^{k_I}\sum_{b=1}^{k_J}\Bigg[& \sum_{l=|n_I-n_J|/2-1}^{(n_I+n_J)/2-2} (2l+1)\ln\sinh\left\{\frac{1}{2}\left(\frac{1}{3}\beta(l+1)+i\left(\alpha_a^{(I)}-\alpha_b^{(J)}\right)\right)\right\} \\
& + \sum_{l=|n_I-n_J|/2+1}^{(n_I+n_J)/2} (2l+1)\ln\sinh\left\{\frac{1}{2}\left(\frac{1}{3}\beta l+i\left(\alpha_a^{(I)}-\alpha_b^{(J)}\right)\right)\right\} \\
& + \sum_{l=|n_I-n_J|/2}^{(n_I+n_J)/2-1} 6(2l+1)\ln\sinh\left\{\frac{1}{2}\left(\frac{1}{3}\beta\left(l+\frac{1}{2}\right)+i\left(\alpha_a^{(I)}-\alpha_b^{(J)}\right)\right)\right\} \\
& - \sum_{l=|n_I-n_J|/2-\frac{1}{2}}^{(n_I+n_J)/2-\frac{3}{2}} 4(2l+1)\ln\cosh\left\{\frac{1}{2}\left(\beta\left(\frac{l}{3}+\frac{1}{4}\right)+i\left(\alpha_a^{(I)}-\alpha_b^{(J)}\right)\right)\right\} \\
& - \sum_{l=|n_I-n_J|/2+\frac{1}{2}}^{(n_I+n_J)/2-\frac{1}{2}} 4(2l+1)\ln\cosh\left\{\frac{1}{2}\left(\beta\left(\frac{l}{3}+\frac{1}{12}\right)+i\left(\alpha_a^{(I)}-\alpha_b^{(J)}\right)\right)\right\}\Bigg]. \quad (3.12)
\end{aligned}
$$

The "ln sinh" and "ln cosh" terms, which come from bosonic and fermionic fluctuations, respectively, yield an attractive potential between the eigenvalues, while the "ln sin" term, which comes from the Vandermonde determinant, yields a repulsive potential.

4. Thermodynamic properties of the plane-wave matrix model

In order to investigate the thermodynamic properties of the plane-wave matrix model, we consider the free energy defined by $F = -T\ln Z - \text{const.}$, where the constant[6] is subtracted in such a way that $\lim_{N\to\infty}\frac{F}{N^2}$ vanishes at $T = 0$. This convention is motivated from the requirement that the free energy F should be of order 1 below the Hagedorn transition [33]. Throughout this section we restrict ourselves to the $s = 1$ case, and use the parameters $n \equiv n_1$ and $k \equiv k_1$.

When we fix k in the large N limit, the $l = O(N)$ terms in the effective action (3.11) have to contribute in order for the free energy to become of order N^2. This, however, does not happen unless $\beta = 0$ since for $\beta \neq 0$ the $l \gtrsim 1/\beta$ terms cancel between bosons and fermions. Therefore, the free energy $\lim_{N\to\infty}\frac{F}{N^2}$ vanishes at any $T < \infty$, which implies that the Hagedorn temperature is infinite [33]. Note also that there are only a finite number of eigenvalues in the present case, and hence there are no critical behaviors associated with the dynamics of the eigenvalues.

On the other hand, when we fix n in the large N limit, the free energy can be of order N^2 at sufficiently high temperature, since the effective action is generically $O(k^2) \sim O(N^2)$. When β is large, the attractive potential is insignificant, and the α_a distribute uniformly in the interval $(-\pi, \pi]$. In this case, we will see that $\lim_{N\to\infty}\frac{F}{N^2}$ actually vanishes. As β decreases, the attractive potential becomes more important. At some point the distribution of α_a becomes non-uniform and $\lim_{N\to\infty}\frac{F}{N^2}$ becomes negative.

[6]This constant is the same for all the classical vacua considered in this paper according to the results of appendix C.

This transition, which is interpreted as the Hagedorn transition [33], is associated with the spontaneous breakdown of the center symmetry $A(t) \mapsto A(t) + \text{const.}\mathbf{1}_N$. The Polyakov line, which is a useful order parameter for the spontaneous symmetry breaking, is given, at the leading order of the perturbation theory, by

$$P \equiv \left\langle \frac{1}{N} \left| \mathrm{tr}\, \mathcal{P} \exp\left(i \int_0^\beta dt \bar{A}(t) \right) \right| \right\rangle = \frac{1}{k} \left\langle \left| \sum_{a=1}^{k} \exp(i\alpha_a) \right| \right\rangle , \qquad (4.1)$$

where the expectation value is taken with respect to the effective action (3.11) for the gauge field moduli. When the center symmetry is unbroken and the distribution of α_a is uniform, we obtain $P = 0$ in the large N limit. When the center symmetry is spontaneously broken, and the distribution of α_a is non-uniform, we obtain $P \neq 0$. This is analogous to the deconfinement phase transition in large N gauge theories [56, 48 − 51, 53].

In what follows we consider the $n = 1, 2, 3$ cases, for which $k = \frac{N}{n}$ goes to infinity with N. The integration over the gauge field moduli, which play a crucial role in the Hagedorn transition, can be done analytically near the transition point. For arbitrary temperature we perform Monte Carlo simulation to confirm the analytical results and to obtain explicit results in the high temperature regime, which is not accessible analytically.

4.1 Analytical results near the transition point

In this section, we investigate the behavior near the Hagedorn transition analytically using the eigenvalue density

$$\rho(\theta) = \frac{1}{N} \sum_{a=1}^{k} \delta(\alpha_a - \theta) , \qquad (4.2)$$

where $\theta \in (-\pi, \pi]$.

First let us review the analysis for the trivial vacuum given in refs. [33, 38]. The effective action (3.9) can be written in terms of the eigenvalue density $\rho(\theta)$ as

$$S_{\text{eff}} = N^2 \int\int d\theta d\theta' \rho(\theta)\rho(\theta') \left\{ - \sum_{m=1}^{\infty} \frac{1}{m} f_m(\beta) e^{im(\theta-\theta')} - \ln \left| \sin\left(\frac{1}{2}(\theta - \theta') \right) \right| \right\} , \qquad (4.3)$$

where we have expanded the "ln sinh", "ln cosh" in (3.11) into Fourier series, and the function $f_m(\beta)$ is defined by

$$f_m(\beta) \equiv 3e^{-\frac{1}{3}\beta m} + 6e^{-\frac{1}{6}\beta m} - 8(-1)^m e^{-\frac{1}{4}\beta m} . \qquad (4.4)$$

By making a Fourier expansion for the eigenvalue density as

$$\rho(\theta) = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} \tilde{\rho}_m e^{im\theta} , \qquad (4.5)$$

the effective action can be rewritten as[7]

$$S_{\text{eff}} = N^2 \sum_{m=1}^{\infty} \frac{1}{m} |\tilde{\rho}_m|^2 \Big\{ 1 - f_m(\beta) \Big\} \,. \tag{4.7}$$

If one of the coefficients of $|\tilde{\rho}_m|^2$ becomes negative, the uniform distribution becomes unstable. This indeed occurs for $m = 1$ as we decrease β from ∞. The Hagedorn temperature $T_H \equiv 1/\beta_H$, which can be determined by solving

$$1 - f_1(\beta_H) = 0\,, \tag{4.8}$$

is $T_H \simeq 0.0758533$.

Since the $\beta_H = 1/T_H$ turned out to be quite large, we may safely omit the $m \geq 2$ terms in the expansion (4.3). This simplification enables the analytic calculation of the thermodynamic quantities by the saddle-point method, which is exact in the large N limit. The saddle-point equation for the eigenvalue density reads

$$0 = \frac{1}{N^2} \frac{d}{d\theta} \frac{\delta S_{\text{eff}}}{\delta \rho(\theta)} \simeq \int d\theta' \rho(\theta') \left\{ f_1(\beta) \sin(\theta - \theta') - \frac{1}{2} \cot\left(\frac{1}{2}(\theta - \theta') \right) \right\} \,. \tag{4.9}$$

This equation can be solved by using the Ansatz of the Gross-Witten form [58]

$$\rho(\theta) = \begin{cases} \frac{2}{\pi\omega} \cos\frac{\theta}{2} \sqrt{\frac{\omega}{2} - \sin^2\frac{\theta}{2}} & (|\theta| \leq \theta_{\text{cl}})\,, \\ 0 & (|\theta| > \theta_{\text{cl}})\,, \end{cases} \tag{4.10}$$

where $\theta_{\text{cl}} = 2\sin^{-1}\sqrt{\frac{\omega}{2}}$ and

$$\omega = 2\left(1 - \sqrt{1 - \frac{1}{f_1(\beta)}} \right) \,. \tag{4.11}$$

Given the eigenvalue density, the free energy[8] and the Polyakov line can be calculated as (See appendix D for the derivation.)

$$\frac{F}{N^2} = -\frac{T S_{\text{eff}}}{N^2} \simeq -T \left\{ \frac{f_1(\beta)}{2} \left(2 - \frac{\omega}{2} \right) + \frac{1}{2} \ln\left(\frac{\omega}{2} \right) - \frac{1}{2} \right\} \,, \tag{4.12}$$

$$P = \int d\theta \rho(\theta) e^{i\theta} \simeq 1 - \frac{\omega}{4} \,. \tag{4.13}$$

The above analysis can be extended to the general n case in a straightforward manner. The only modifications in the effective action (4.3) are to replace the overall factor N^2 by k^2, and to replace the functions $f_m(\beta)$ by

$$f_m(\beta) = \sum_{l=0}^{n-2} (2l+1) e^{-\frac{1}{3}\beta m(l+1)} + \sum_{l=1}^{n} (2l+1) e^{-\frac{1}{3}\beta m l} + \sum_{l=0}^{n-1} 6(2l+1) e^{-\frac{1}{3}\beta m \left(l + \frac{1}{2}\right)}$$

[7] In arriving at (4.7), we have also expanded the "ln sin" term using the formula

$$\ln \sin\frac{\theta}{2} = -\sum_{m=1}^{\infty} \frac{1}{m} \cos m\theta + \text{const.} \tag{4.6}$$

[8] We have corrected an error in the analytic expression for the free energy given in ref. [38].

$$
-\sum_{l=\frac{1}{2}}^{n-\frac{3}{2}}(-1)^m 4(2l+1)e^{-\beta m(\frac{l}{3}+\frac{1}{4})} - \sum_{l=\frac{1}{2}}^{n-\frac{1}{2}}(-1)^m 4(2l+1)e^{-\beta m(\frac{l}{3}+\frac{1}{12})} \ . \qquad (4.14)
$$

By solving eq. (4.8) in the present case, we obtain the Hagedorn temperature

$$
T_H \simeq \begin{cases} 0.0738901 & (n=2) \\ 0.0738526 & (n=3) \\ 0.0738520 & (n \geq 4) \ . \end{cases} \qquad (4.15)
$$

Since these values are small, we may neglect the $m \geq 2$ terms in (4.3) in the general case as well. The expression for the free energy (4.12) gets multiplied by the factor $\frac{1}{n^2}$ due to the modified prefactor in S_{eff} mentioned above, while the expression for the Polyakov line (4.13) remains the same. Note that one has to use $f_1(\beta)$ given by (4.14) for the definition of ω in eq. (4.11). In section 4.3 we will confirm the validity of these analytical results by Monte Carlo simulation.

4.2 High temperature limit

At high temperature (i.e., small β), the "$\ln\sinh$" terms in the effective action (3.11) make the eigenvalues attracted to each other against the repulsive force coming from the "$\ln\sin$" terms, and therefore $(\alpha_a - \alpha_b)$ is typically of order β. The free energy can therefore be estimated by simply replacing the "$\ln\sinh$" and "$\ln\sin$" terms by $\ln\beta$, and omitting the "$\ln\cosh$" terms in the effective action (3.11). This gives

$$
\frac{F}{N^2} = -\left(8 + \frac{1}{nN}\right) T\ln T + O(T) \ . \qquad (4.16)
$$

Thus, the leading asymptotic behavior of $\lim_{N\to\infty}\frac{F}{N^2}$ at high temperature is universal. In section 4.3 we will confirm this asymptotic behavior by Monte Carlo simulation.

4.3 Monte Carlo integration over the gauge field moduli

In this section we perform Monte Carlo simulation to integrate over the gauge field moduli, and obtain explicit results for the free energy and the Polyakov line at arbitrary temperature.

We use the Metropolis algorithm for the simulation. At each step we generate a trial configuration by replacing one of the eigenvalues by a random number within the interval $(-\pi,\pi]$, and accept it with the probability $\max(1, e^{-\Delta S_{\text{eff}}})$, where ΔS_{eff} is the difference of the effective action (3.11) for the trial configuration from that for the previous configuration.

In figure 1 we plot the expectation value of the Polyakov line near the transition point. Our Monte Carlo results are in good agreement with the analytic results (4.13) including the position of the transition point. We have also made a similar plot for the $n=3$ case, but it turned out to be almost indistinguishable from the results for the $n=2$ case. In figures 2 and 3 we plot the distribution of the eigenvalues. While our data agree with the analytic results (4.10) near the transition point, we also start to see a small deviation as the temperature increases.

The free energy can be calculated by

$$F = -T\langle S_{\text{eff}} \rangle \tag{4.17}$$

in the large N limit. In order to obtain the free energy accurately near the critical point, we have to make a large N extrapolation. The dominant finite N effects come from the "ln sin" term in the effective action (3.11) due to its logarithmic singularity when the eigenvalues come close to each other. Since the distance between the nearest eigenvalues is of order $\frac{1}{N}$, the finite N effects for evaluating the $O(1)$ quantity

$$\frac{1}{N^2}\sum_{a\neq b}^{N} \ln\left|\sin\frac{\alpha_a-\alpha_b}{2}\right| ,$$

for instance, is $O\left(\frac{\ln N}{N}\right)$. In figure 4 we therefore plot the free energy obtained by (4.17) at finite N against $\frac{\ln N}{N}$ in the trivial vacuum case for various temperature. Indeed our data can be nicely fitted to straight lines, from which we can make a reliable large-N extrapolation. The large-N limits obtained in this way are plotted in figure 5 against the temperature T near the critical point. Our results agree nicely to the analytic results (4.12). At sufficiently high temperature, we observe that the trivial vacuum gives the smallest free energy. As we decrease the temperature, however, it is taken over by the $n = 2$ vacuum.

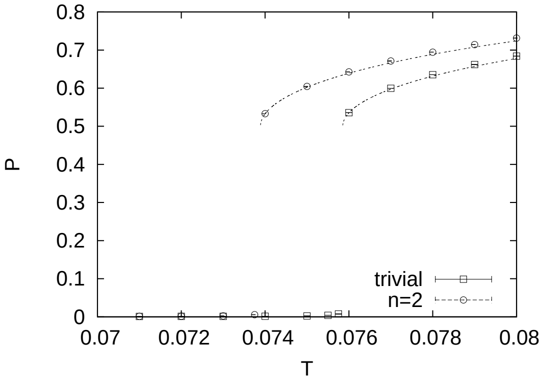

Figure 1: The Polyakov line P is plotted against the temperature T near the critical point for $N = 2000$ in the trivial case, and for $k = 2000$ in the $n = 2$ case. The dotted lines represent the analytic results (4.13).

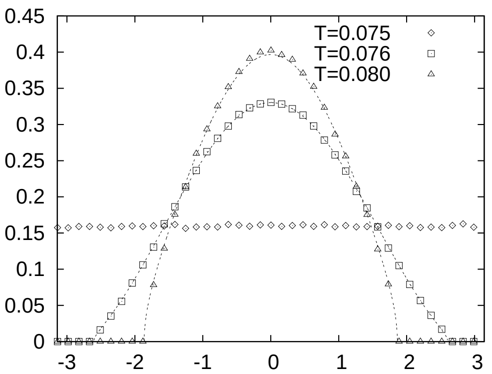

Figure 2: The eigenvalue distribution $\rho(\theta)$ in the trivial vacuum case is plotted for $T = 0.075$, 0.076, 0.080 and $N = 2000$. The dashed lines represent the analytic result (4.10) obtained with the Ansatz of the Gross-Witten form.

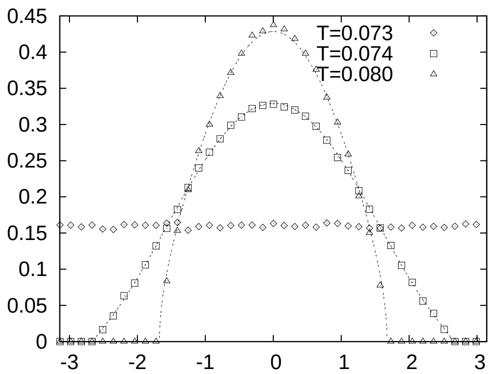

Figure 3: The eigenvalue distribution $\rho(\theta)$ in the $n = 2$ case is plotted for $T = 0.073$, 0.074, 0.080 and $k = 2000$. The dashed lines represent the analytic result (4.10) obtained with the Ansatz of the Gross-Witten form.

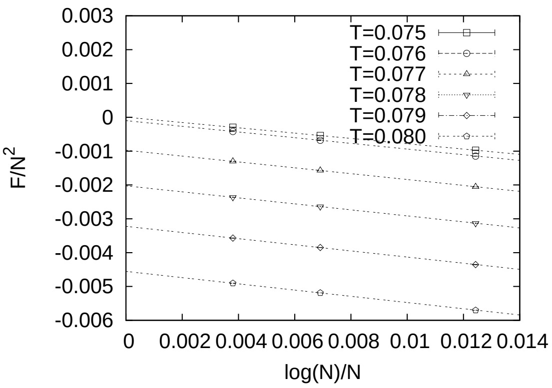

Figure 4: The free energy F is plotted against $\frac{\ln N}{N}$ for $N = 500$, 1000 and 2000 in the trivial vacuum case for various temperature. Our data are nicely fitted to straight lines.

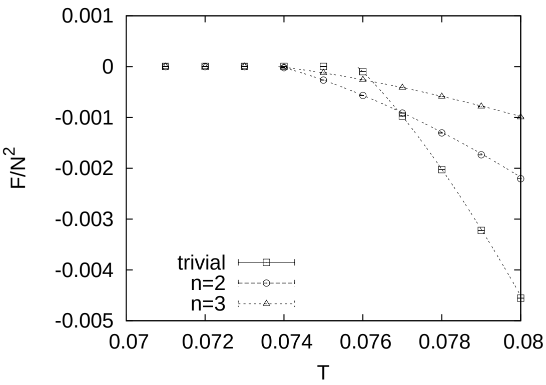

Figure 5: The free energy F obtained by the large N extrapolation is plotted against the temperature T near the critical point for the trivial vacuum as well as for the $n = 2, 3$ cases. The curves represent the analytic results (4.12).

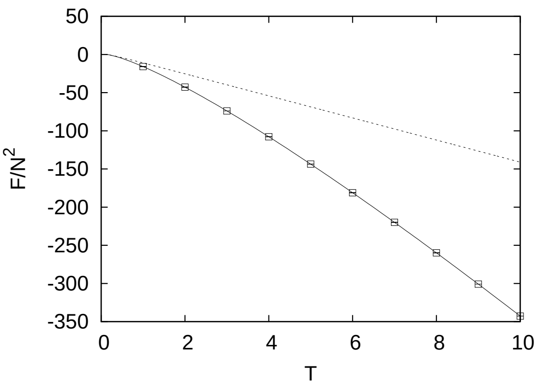

Figure 6: The free energy $\frac{F}{N^2}$ is plotted against T for $N = 500$ in the trivial vacuum case. The solid line represents the result for fitting the data for $1 \leq T \leq 10$ to (4.18), where $c_1 = 15.852(1)$ and $c_2 = 0.038(3)$. The dotted line represents the analytic result for the free energy (4.12).

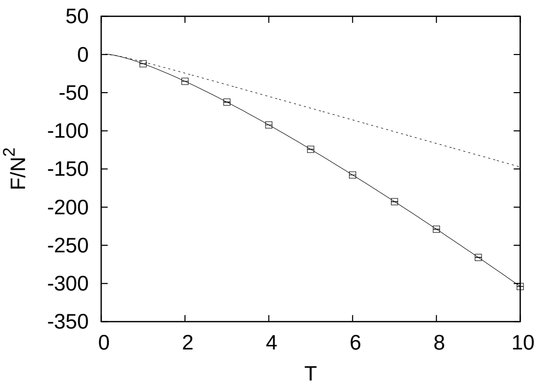

Figure 7: The free energy $\frac{F}{N^2}$ is plotted against T for $n = 2$ and $k = 500$. The solid line represents the result for fitting the data for $1 \leq T \leq 10$ to (4.18), where $c_1 = 11.948(5)$ and $c_2 = 0.16(1)$. The dotted line represents the analytic result for the free energy (4.12).

In figures 6 and 7 we plot the free energy $\frac{F}{N^2}$ at higher temperature. The deviation from the analytic result for the free energy (4.12) becomes pronounced for $T \gtrsim 1$ as expected, and our data can be nicely fitted to

$$\frac{F}{N^2} \simeq -8T\log(T) - c_1 T - c_2 \,, \tag{4.18}$$

where the leading asymptotic behavior is determined analytically in section 4.2. Although the leading term is the same at $N = \infty$, the coefficient c_1 of the linear term turns out to be larger for the trivial vacuum case. Hence we conclude that the trivial vacuum gives the smallest free energy for $T \gtrsim 0.077$.

5. Summary and discussions

We have studied the thermodynamic properties of the plane-wave matrix model in general classical vacua. The Hagedorn transition occurs due to the dynamics of the gauge field moduli. Extending the previous works on the trivial vacuum, we obtained analytical results for the thermodynamic quantities near the transition point, which revealed the existence of the "fuzzy-sphere" phase. We also performed Monte Carlo simulation to integrate over the gauge field moduli. This method can be used for arbitrary temperature. Usually free energy is difficult to calculate by Monte Carlo simulation, but in the present case we can obtain it very accurately from the expectation value of the effective action for the gauge field moduli, thanks to large N. We observe that the trivial vacuum gives the smallest free energy in the high temperature regime.

The plane-wave matrix model is closely related to $\mathcal{N}=4$ SYM theory on $R\times S^3$ [59], which implies that the matrix model at finite temperature is related to the SYM theory on $S^1\times S^3$. We therefore expect that our results should have implications on the phase structure of the SU(∞) SYM in four dimensions. In particular, the fuzzy sphere phases we found in this paper may correspond to the plasma-ball phases in the SU(∞) SYM on $S^1\times S^3$, which are interpreted as localized black holes on the gravity side via the AdS/CFT duality [41]. Another interesting future direction is to clarify the relation between our result and the entropy bound discussed recently [60].

We hope that our work provides a clue to the phase structure of the AdS black hole and to a deeper understanding of the AdS/CFT correspondence at finite temperature.

Acknowledgments

We would like to thank T. Azuma, H. Fuji, Y. Hikida, S. Mizoguchi, Y. Susaki, S. Takeuchi and A. Yamaguchi for helpful discussion, and G. Semenoff for explaining his works on this subject. K.Y. is grateful to H. Shin for a series of fruitful collaborations, which are closely related to the topic discussed in this paper. The work of K.Y. is supported in part by JSPS Research Fellowships for Young Scientists.

Appendix

A. Gauge fixing

In order to arrive at our main result eq. (3.8) in section 3, we have first integrated over the fluctuations fixing the gauge field moduli, and then the integration over the gauge field moduli has been written in terms of the moduli parameters $\alpha_a^{(I)}$. Since the classical vacua (2.4) breaks the U(N) gauge symmetry down to $\prod_{I=1}^{s}\mathrm{U}(k_I)$, we have to make gauge fixing at each step of the above procedure. First, when we integrate over the fluctuations, we have to fix the gauge along the gauge orbit in the direction of the coset space $\frac{\mathrm{U}(N)}{\prod_{I=1}^{s}\mathrm{U}(k_I)}$.

This is necessary since otherwise there will be zero modes corresponding to the broken symmetry, and we cannot integrate over the fluctuations. Next, when we integrate over the gauge field moduli, we have to fix the gauge corresponding to the remaining symmetry $\prod_{I=1}^{s} \mathrm{U}(k_I)$ in order to reduce the integration to that over the moduli parameters $\alpha_a^{(I)}$. This is necessary for studying the large N limit analytically by the eigenvalue density method and for simplifying Monte Carlo calculations drastically. In what follows we explain the two steps of gauge fixing procedures separately.

A.1 Integrating the fluctuations

Let us explain the first gauge fixing, which is necessary for integrating the fluctuations. We use the background field gauge

$$\overline{D}_\mu A_\mu(t) = r(t)\,, \tag{A.1}$$

where $r(t)$ is an arbitrary function, and $\overline{D}_\mu$ and $A_\mu(t)$ for $\mu = t, 1, 2, 3$ are defined by

$$\overline{D}_t = \partial_t - i[\bar{A}(t),\ \cdot\ ]\,, \quad A_t(t) = A(t)\,, \tag{A.2}$$

$$\overline{D}_i = -i[B_i,\ \cdot\ ]\,, \qquad A_i(t) = X_i(t) \qquad \text{for } i = 1, 2, 3\,. \tag{A.3}$$

The corresponding Faddeev-Popov (FP) determinant is defined by

$$1 = \int dg(t) \delta\left(\overline{D}_\mu A_\mu^g(t) - r(t)\right) \Delta_{\mathrm{FP}}^{(1)}\,, \tag{A.4}$$

where $g(t)$ is an element of the coset group $\frac{\mathrm{U}(N)}{\prod_{I=1}^{s}\mathrm{U}(k_I)}$ and $A_\mu^g(t)$ represents the gauge transformed field. The FP determinant can be represented as

$$\Delta_{\mathrm{FP}}^{(1)} = \int [dc][d\bar{c}] \exp\left(-\int dt \ \mathrm{tr}\left(\overline{D}_t\bar{c} \cdot D_t c - [B_i, \bar{c}][X_i, c]\right)\right)\,, \tag{A.5}$$

where c, $\bar{c}$ are the ghost fields satisfying the periodic boundary condition. Inserting the identity (A.4) in the partition function, and integrating over $r(t)$ with the gaussian weight $\exp\left(-\frac{1}{2}\int dt\, r^2(t)\right)$, we obtain the gauge fixing term and the ghost term

$$S_{\mathrm{g.f.}} = \int dt \ \mathrm{tr}\left\{\frac{1}{2}\left(\overline{D}_\mu A_\mu\right)^2\right\}\,, \quad S_{\mathrm{ghost}} = \int dt \ \mathrm{tr}\left\{\overline{D}_t\bar{c}\cdot D_t c - [B_i, \bar{c}][X_i, c]\right\}\,, \tag{A.6}$$

which should be added to the original action (2.1).

A.2 Integrating the gauge field moduli

Let us explain the second gauge fixing, which is necessary for integrating the gauge field moduli. We use the static diagonal gauge, which is analogous to the one adopted for the trivial vacuum [33].

First we impose the static gauge condition given by

$$\partial_t \bar{A}^{(I)}(t) = 0\ . \tag{A.7}$$

The corresponding FP determinant can be defined by

$$1 = \int \prod_{I=1}^{s} d\tilde{g}^{(I)}(t)\delta\left(\frac{d\bar{A}^{(I)}(t)}{dt}\right)\Delta_{\rm FP}^{(2)}\,, \tag{A.8}$$

where $\tilde{g}^{(I)}(t)$ is an element of the unbroken gauge group $\prod_{I=1}^{s}\mathrm{U}(k_I)$. The FP determinant $\Delta_{\rm FP}^{(2)}$ is given by

$$\Delta_{\rm FP}^{(2)} = \prod_{I=1}^{s}\det{}'\left(\partial_t\overline{D}_t^{(I)}\right)\,, \tag{A.9}$$

where we have defined $\overline{D}_t^{(I)} \equiv \partial_t - i[\bar{A}^{(I)},\ \cdot\ ]$, and the symbol $\det'$ implies that we have omitted the zero modes. The static gauge (A.7) does not fix the gauge completely, and we still have the global $\prod_{I=1}^{s}\mathrm{U}(k_I)$ symmetry as the residual gauge symmetry, which we fix by further imposing the constant matrices $\bar{A}_{ab}^{(I)}$ to be diagonal (3.2).

As is well known in matrix models, this gauge fixing yields the Vandermonde determinant, which is derived as follows. The path integral measure for the gauge field moduli $[d\bar{A}(t)]$ around the static diagonal configuration (3.2) is rewritten as

$$[d\bar{A}(t)] = \prod_{I=1}^{s}\left(\frac{T^{k_I^2}}{k_I!}\cdot\prod_{a=1}^{k_I}\frac{d\alpha_a^{(I)}}{2\pi}\cdot\prod_{a\neq b}^{k_I}\left|\alpha_a^{(I)}-\alpha_b^{(I)}\right|\cdot dg^{(I)}\cdot\prod_{m\neq 0}\prod_{a,b}^{k_I}d\bar{A}_{m,ab}^{(I)}\right)\,, \tag{A.10}$$

where $g^{(I)}$ is an element of $\prod_{I=1}^{s}\mathrm{U}(k_I)$, and $d\bar{A}_m^{(I)}$ $(m\neq 0)$ is the integration measure for the non-zero Fourier modes.

Let us evaluate the determinant (A.9) explicitly. In terms of Fourier modes,[9] the FP determinant (A.9) reads

$$\Delta_{\rm FP}^{(2)} = \prod_{I=1}^{s}\left[\prod_{a,b=1}^{k_I}\prod_{m\neq 0}(2\pi imT)\Big\{2\pi imT + i(\alpha_a^{(I)}-\alpha_b^{(I)})T\Big\}\right]\,. \tag{A.12}$$

Therefore the identity (A.8) is rewritten as

$$\begin{aligned}
1 &= \prod_{I=1}^{s}\left[d\tilde{g}^{(I)}(t)\prod_{a,b=1}^{k_I}\prod_{m\neq 0}\delta(2\pi imT\bar{A}_m^{(I)})(2\pi imT)\Big\{2\pi imT + i(\alpha_a^{(I)}-\alpha_b^{(I)})T\Big\}\right] \\
&= \prod_{m\neq 0}(2\pi imT)^{\sum_{I=1}^{s}k_I^2}\cdot\prod_{I=1}^{s}\left[d\tilde{g}^{(I)}(t)\prod_{a\neq b}^{k_I}\prod_{m=1}^{\infty}\left\{1-\left(\frac{\alpha_a^{(I)}-\alpha_b^{(I)}}{2\pi m}\right)^2\right\}\right]\,.
\end{aligned} \tag{A.13}$$

[9] The fields $M(t)$ and $\Theta(t)$ obeying the periodic and anti-periodic boundary conditions, are expanded as

$$M(t) = \frac{1}{\sqrt{\beta}}\sum_{m=-\infty}^{\infty}M_m\mathrm{e}^{\frac{2\pi im}{\beta}t}\,,\qquad \Theta(t) = \frac{1}{\sqrt{\beta}}\sum_{m=-\infty}^{\infty}\Theta_m\mathrm{e}^{\frac{\pi i(2m-1)}{\beta}t}\,, \tag{A.11}$$

respectively.

In the second line, we have omitted the delta function for the non-zero Fourier modes, which are integrated out by using the measure in (A.10). Inserting the identity (A.13) in the partition function, the measure (A.10) for the gauge field moduli $[d\bar{A}(t)]$ can be written as

$$[d\bar{A}(t)] = (2T)^{\sum_{I=1}^{s} k_I^2} \cdot \prod_{m\neq 0} (2\pi i m T)^{\sum_{I=1}^{s} k_I^2} \times \prod_{I=1}^{s} \left\{ dg^{(I)}(t) \cdot \frac{1}{k_I!} \prod_{a=1}^{k_I} \frac{d\alpha_a^{(I)}}{4\pi} \cdot \prod_{a\neq b}^{k_I} \left| \sin \frac{\alpha_a^{(I)} - \alpha_b^{(I)}}{2} \right| \right\}, \qquad \text{(A.14)}$$

where $dg^{(I)}(t) \equiv d\tilde{g}^{(I)}(t) \cdot dg^{(I)}$ represents the measure for the gauge function of the $\prod_{I=1}^{s} \mathrm{U}(k_I)$ symmetry. The "sin" term in (A.14) appeared from the formula used to sum over the Fourier modes

$$\prod_{n=1}^{\infty} \left(1 - \frac{x^2}{n^2}\right) = \frac{\sin \pi x}{\pi x}.$$

The result for the trivial vacuum can be obtained formally by setting $s = 1$, $k_1 = N$ and $n_1 = 1$, which yields

$$[d\bar{A}(t)] = (2T)^{N^2} \cdot \prod_{m\neq 0} (2\pi i m T)^{N^2} \cdot dg(t) \cdot \frac{1}{N!} \prod_{a=1}^{N} \frac{d\alpha_a}{4\pi} \cdot \prod_{a\neq b}^{N} \left| \sin \frac{\alpha_a - \alpha_b}{2} \right|, \qquad \text{(A.15)}$$

where $g(t)$ represents the gauge function of the $\mathrm{U}(N)$ symmetry. This agrees with the measure obtained in ref. [33].

B. Evaluation of the determinants

Integrating over the fluctuation Y, Ψ, $\tilde{A}$ and ghost fields using the quadratic action (3.6), we obtain the determinants

$$\begin{aligned} Y, \tilde{A} :& \quad \det{}_{\mathrm{B}}^{-1/2}(\overline{D}_t^2 + \lambda^2), \\ \Psi :& \quad \det{}_{\mathrm{F}}(\overline{D}_t + \lambda), \\ c, \bar{c} :& \quad \det{}_{\mathrm{B}}(\overline{D}_t^2 + \lambda^2), \end{aligned} \qquad \text{(B.1)}$$

where λ represents an eigenvalue of the mass operators (3.7), and the lower suffices of det specify the boundary condition (periodic for "B" and anti-periodic for "F") for the fields on which the operators act.

Using the formulae

$$\prod_{n=1}^{\infty} \left(1 + \frac{x^2}{n^2}\right) = \frac{\sinh \pi x}{\pi x}, \qquad \prod_{n=1}^{\infty} \left(1 + \frac{x^2}{(2n-1)^2}\right) = \cosh\left(\frac{\pi x}{2}\right),$$

we obtain the determinants as

$$\det{}_{\mathrm{B}}^{1/2} \left(-\overline{D}_t^2 + \lambda^2\right) = \prod_{m} \prod_{a,b} \{2\pi i m T + i(\alpha_a - \alpha_b)T + \lambda\}$$

	Gaussian	determinant	gauge volume
$\int dg(t)$			$\left(\frac{\mathrm{Vol}(\mathrm{U}(N))}{\prod_{I=1}^{s}\mathrm{Vol}(\mathrm{U}(k_I))}\right)_{\mathrm{local}}$
$\int \prod_I^s dg^{(I)}(t)$			$\prod_{I=1}^{k_I}\mathrm{Vol}(\mathrm{U}(k_I))_{\mathrm{local}}$
$r(t)$-int.	$(\prod_m 2\pi)^{-\frac{1}{2}(N^2-\sum_{I=1}^{s}k_I^2)}$		
$Y(t)$-int.	$(\prod_m 2\pi)^{\frac{9}{2}N^2}$	$f(T)^{-9N^2}$	
$\tilde{A}(t)$-int.	$(\prod_m 2\pi)^{\frac{1}{2}(N^2-\sum_{I=1}^{s}k_I^2)}$	$f(T)^{-(N^2-\sum_{I=1}^{s}k_I^2)}$	
$\Psi(t)$-int.		$g(T)^{8N^2}$	
$c(t)$, $\bar{c}(t)$-int.		$f(T)^{2(N^2-\sum_{I=1}^{s}k_I^2)}$	
$[d\tilde{A}]$		$f(T)^{\sum_{I=1}^{s}k_I^2}$	
$\mathcal{C}$	$(\prod_m 2\pi)^{\frac{9}{2}N^2}$	$f(T)^{-8N^2}\cdot g(T)^{8N^2}$	$\mathrm{Vol}(\mathrm{U}(N))_{\mathrm{local}}$

Table 3: The list of contributions to the overall factor of the partition function for general classical vacua.

$$= (2T)^{N^2}\cdot\prod_{m\neq 0}(2\pi i m T)^{N^2}\cdot\prod_{a,b}\sinh\left\{\frac{1}{2}\left(\beta\lambda+i(\alpha_a-\alpha_b)\right)\right\}\ , \quad \text{(B.2)}$$

$$\det{}_{\mathrm{F}}\left(\overline{D}_t+\lambda\right) = \prod_m\prod_{a,b}\left\{\pi i(2m-1)T+i(\alpha_a-\alpha_b)T+\lambda\right\}$$
$$= \prod_m(\pi i(2m-1)T)^{N^2}\cdot\prod_{a,b}\cosh\left\{\frac{1}{2}\left(\beta\lambda+i(\alpha_a-\alpha_b)\right)\right\}\ . \quad \text{(B.3)}$$

The coefficients in front of "sinh" and "cosh" have to be taken into account in comparing the free energy for different vacua.

C. The overall factor of the partition function

In this section we obtain the overall factor $\mathcal{C}$ in (3.8). There are three types of contributions to $\mathcal{C}$, which affect the calculation of free energy at the leading order in N; namely the Gaussian integration, the determinants, and the gauge volume obtained from gauge fixing. The factor obtained from each contribution is given in table 3. The symbol $\mathrm{Vol}(\mathrm{U}(N))_{\mathrm{local}}$ represents the gauge volume for the local $\mathrm{U}(N)$ symmetry, and we have introduced the functions

$$f(T)\equiv 2T\cdot\prod_{m\neq 0}(2\pi i m T)\ ,\quad g(T)\equiv\prod_m\left\{\pi i(2m-1)T\right\}\ .$$

The overall factor of the partition function $\mathcal{C}$ turns out to the same for all the classical vacua, and it is given by

$$\mathcal{C}\equiv\left(\prod_m 2\pi\right)^{\frac{9}{2}N^2}\cdot f(T)^{-8N^2}\cdot g(T)^{8N^2}\cdot\mathrm{Vol}(\mathrm{U}(N))_{\mathrm{local}}\ . \quad \text{(C.1)}$$

JHEP06(2006)052

D. Evaluating free energy with the eigenvalue density

In this section we derive the free energy (4.12) for the eigenvalue density (4.10). Omitting the $m \geq 2$ terms in (4.3), the effective action is given as

$$
\begin{aligned}
S_{\rm eff} \approx & -N^2 \int_{-\theta_{\rm cl}}^{\theta_{\rm cl}} \int_{-\theta_{\rm cl}}^{\theta_{\rm cl}} d\theta d\theta' \rho(\theta)\rho(\theta') f_1(\beta) \cos(\theta - \theta') \\
& -N^2 \mathcal{P}.\mathcal{V}. \int_{-\theta_{\rm cl}}^{\theta_{\rm cl}} \int_{-\theta_{\rm cl}}^{\theta_{\rm cl}} d\theta d\theta' \rho(\theta)\rho(\theta') \ln \left| \sin\left(\frac{1}{2}(\theta - \theta')\right) \right| \\
& +N^2 \frac{1}{2\pi} \int_{-\pi}^{\pi} d\theta \ln \left| \sin \frac{\theta}{2} \right| ,
\end{aligned} \tag{D.1}
$$

where the symbol $\mathcal{P}.\mathcal{V}.$ represents the principal value. For eigenvalue density (4.10) one can easily obtain

$$
\int_{-\theta_{\rm c}}^{\theta_{\rm cl}} d\theta \rho(\theta) \sin\theta = 0 \,, \quad \int_{-\theta_{\rm c}}^{\theta_{\rm cl}} d\theta \rho(\theta) \cos\theta = 1 - \frac{\omega}{4} \,, \quad \int_{-\pi}^{\pi} d\theta \ln \left| \sin \frac{\theta}{2} \right| = -\ln 2 \,, \tag{D.2}
$$

$$
\mathcal{P}.\mathcal{V}. \int_{-\theta_{\rm cl}}^{\theta_{\rm cl}} d\theta d\theta' \rho(\theta)\rho(\theta') \ln \left| \sin \frac{\theta - \theta'}{2} \right| = -\ln 2 + \frac{1}{2} \ln \frac{\omega}{2} - \frac{1}{4} \,, \tag{D.3}
$$

where, in deriving the formula in the second line, we have used the fact that the distribution (4.10) satisfies the saddle-point equation (4.9). Using these formulae, we obtain

$$
-\frac{S_{\rm eff}}{N^2} \approx f_1(\beta) \left(1 - \frac{\omega}{4}\right)^2 + \frac{1}{2} \ln \frac{\omega}{2} - \frac{1}{4} = \frac{f_1(\beta)}{2} \left(2 - \frac{\omega}{2}\right) + \frac{1}{2} \ln \frac{\omega}{2} - \frac{1}{2} \,. \tag{D.4}
$$

References

[1] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, *M theory as a matrix model: a conjecture*, *Phys. Rev.* **D 55** (1997) 5112 [hep-th/9610043].

[2] N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, *A large-N reduced model as superstring*, *Nucl. Phys.* **B 498** (1997) 467 [hep-th/9612115].

[3] R. Dijkgraaf, E. Verlinde and H. Verlinde, *Matrix string theory*, *Nucl. Phys.* **B 500** (1997) 43 [hep-th/9703030].

[4] B. de Wit, J. Hoppe and H. Nicolai, *On the quantum mechanics of supermembranes*, *Nucl. Phys.* **B 305** (1988) 545.

[5] I.R. Klebanov and L. Susskind, *Schwarzschild black holes in various dimensions from matrix theory*, *Phys. Lett.* **B 416** (1998) 62 [hep-th/9709108].

[6] T. Banks, W. Fischler, I.R. Klebanov and L. Susskind, *Schwarzschild black holes in matrix theory, II*, *JHEP* **01** (1998) 008 [hep-th/9711005].

[7] D. Kabat, G. Lifschytz and D.A. Lowe, *Black hole thermodynamics from calculations in strongly coupled gauge theory*, *Phys. Rev. Lett.* **86** (2001) 1426 [*Int. J. Mod. Phys.* **A 16** (2001) 856] [hep-th/0007051].

[8] J. Ambjørn, Y.M. Makeenko and G.W. Semenoff, *Thermodynamics of D0-branes in matrix theory Phys. Lett.* **B 445** (1999) 307 [hep-th/9810170];
Y. Makeenko, *Formulation of matrix theory at finite temperature*, *Fortschr. Phys.* **48** (2000) 171 [hep-th/9903030].

JHEP06(2006)052

[9] M. Li, *Ten dimensional black hole and the D0-brane threshold bound state*, *Phys. Rev.* **D 60** (1999) 066002 [hep-th/9901158].
S. Bal and B. Sathiapalan, *High temperature limit of the $N = 2$ matrix model*, *Mod. Phys. Lett.* **A 14** (1999) 2753 [hep-th/9902087]; *High temperature limit of the $N = 2$ IIA matrix model*, *Nucl. Phys.* **94** *(Proc. Suppl.)* (2001) 693 [hep-lat/0011039].

[10] J.L. Karczmarek and A. Strominger, *Matrix cosmology*, *JHEP* **04** (2004) 055 [hep-th/0309138];
S.R. Das, J.L. Davis, F. Larsen and P. Mukhopadhyay, *Particle production in matrix cosmology*, *Phys. Rev.* **D 70** (2004) 044017 [hep-th/0403275];
D.Z. Freedman, G.W. Gibbons and M. Schnabl, *Matrix cosmology*, *AIP Conf. Proc.* **743** (2005) 286 [hep-th/0411119].

[11] J. Kowalski-Glikman, *Vacuum states in supersymmetric Kaluza-Klein theory*, *Phys. Lett.* **B 134** (1984) 194.

[12] D. Berenstein, J.M. Maldacena and H. Nastase, *Strings in flat space and pp waves from $N = 4$ super Yang Mills*, *JHEP* **04** (2002) 013 [hep-th/0202021].

[13] K. Dasgupta, M.M. Sheikh-Jabbari and M. Van Raamsdonk, *Matrix perturbation theory for M-theory on a PP-wave*, *JHEP* **05** (2002) 056 [hep-th/0205185].

[14] K. Sugiyama and K. Yoshida, *Supermembrane on the pp-wave background*, *Nucl. Phys.* **B 644** (2002) 113 [hep-th/0206070]; *BPS conditions of supermembrane on the pp-wave*, *Phys. Lett.* **B 546** (2002) 143 [hep-th/0206132].

[15] N. Nakayama, K. Sugiyama and K. Yoshida, *Ground state of the supermembrane on a pp-wave*, *Phys. Rev.* **D 68** (2003) 026001 [hep-th/0209081].

[16] R.C. Myers, *Dielectric-branes*, *JHEP* **12** (1999) 022 [hep-th/9910053].

[17] K. Sugiyama and K. Yoshida, *Giant graviton and quantum stability in matrix model on PP-wave background*, *Phys. Rev.* **D 66** (2002) 085022 [hep-th/0207190].

[18] H. Shin and K. Yoshida, *One-loop flatness of membrane fuzzy sphere interaction in plane-wave matrix model*, *Nucl. Phys.* **B 679** (2004) 99 [hep-th/0309258].

[19] K. Dasgupta, M.M. Sheikh-Jabbari and M. Van Raamsdonk, *Protected multiplets of M-theory on a plane wave*, *JHEP* **09** (2002) 021 [hep-th/0207050].

[20] N. Kim and J. Plefka, *On the spectrum of pp-wave matrix theory*, *Nucl. Phys.* **B 643** (2002) 31 [hep-th/0207034].

[21] N. Kim and J.H. Park, *Superalgebra for M-theory on a pp-wave*, *Phys. Rev.* **D 66** (2002) 106007 [hep-th/0207061].

[22] D. Bak, *Supersymmetric branes in PP wave background*, *Phys. Rev.* **D 67** (2003) 045017 [hep-th/0204033].

[23] S. Hyun and H. Shin, *Branes from matrix theory in pp-wave background*, *Phys. Lett.* **B 543** (2002) 115 [hep-th/0206090].

[24] J.H. Park, *Supersymmetric objects in the M-theory on a pp-wave*, *JHEP* **10** (2002) 032 [hep-th/0208161].

[25] D. Bak, S. Kim and K. Lee, *All higher genus BPS membranes in the plane wave background*, *JHEP* **06** (2005) 035 [hep-th/0501202].

JHEP06(2006)052

[26] H. Shin and K. Yoshida, *Membrane fuzzy sphere dynamics in plane-wave matrix model*, *Nucl. Phys.* **B 709** (2005) 69 [hep-th/0409045].

[27] K. Sugiyama and K. Yoshida, *Type IIA string and matrix string on pp-wave*, *Nucl. Phys.* **B 644** (2002) 128 [hep-th/0208029].

[28] S.R. Das, J. Michelson and A.D. Shapere, *Fuzzy spheres in pp-wave matrix string theory*, *Phys. Rev.* **D 70** (2004) 026004 [hep-th/0306270].

[29] S.R. Das and J. Michelson, *pp wave big bangs: matrix strings and shrinking fuzzy spheres*, *Phys. Rev.* **D 72** (2005) 086005 [hep-th/0508068].

[30] H. Shin and K. Yoshida, *Point-like graviton scattering in plane-wave matrix model*, *JHEP* **04** (2006) 051 [hep-th/0511072].

[31] H. Shin and K. Yoshida, *Graviton and spherical graviton potentials in plane-wave matrix model: overview and perspective*, hep-th/0511187.

[32] T. Kimura and K. Yoshida, *Spectrum of eleven-dimensional supergravity on a pp-wave background*, *Phys. Rev.* **D 68** (2003) 125007 [hep-th/0307193].

[33] K. Furuuchi, E. Schreiber and G.W. Semenoff, *Five-brane thermodynamics from the matrix model*, hep-th/0310286.

[34] J. Maldacena, M.M. Sheikh-Jabbari and M. Van Raamsdonk, *Transverse fivebranes in matrix theory*, *JHEP* **01** (2003) 038 [hep-th/0211139].

[35] Y. Lozano and D. Rodriguez-Gomez, *Fuzzy 5-spheres and pp-wave matrix actions*, *JHEP* **08** (2005) 044 [hep-th/0505073].

[36] M. Spradlin, M. Van Raamsdonk and A. Volovich, *Two-loop partition function in the planar plane-wave matrix model*, *Phys. Lett.* **B 603** (2004) 239 [hep-th/0409178].

[37] S. Hadizadeh, B. Ramadanovic, G.W. Semenoff and D. Young, *Free energy and phase transition of the matrix model on a plane-wave*, *Phys. Rev.* **D 71** (2005) 065016 [hep-th/0409318].

[38] G.W. Semenoff, *Matrix model thermodynamics*, hep-th/0405107.

[39] W.H. Huang, *Thermal instability of giant graviton in matrix model on pp-wave background*, *Phys. Rev.* **D 69** (2004) 067701 [hep-th/0310212].

[40] H. Shin and K. Yoshida, *Thermodynamics of fuzzy spheres in pp-wave matrix model*, *Nucl. Phys.* **B 701** (2004) 380 [hep-th/0401014];
Thermodynamic behavior of fuzzy membranes in PP-wave matrix model, *Phys. Lett.* **B 627** (2005) 188 [hep-th/0507029].

[41] O. Aharony, S. Minwalla and T. Wiseman, *Plasma-balls in large N gauge theories and localized black holes*, hep-th/0507219.

[42] T. Azuma, S. Bal and J. Nishimura, *Dynamical generation of gauge groups in the massive Yang-Mills-Chern-Simons matrix model*, *Phys. Rev.* **D 72** (2005) 066005 [hep-th/0504217].

[43] T. Azuma, S. Bal, K. Nagao and J. Nishimura, *Nonperturbative studies of fuzzy spheres in a matrix model with the Chern-Simons term*, *JHEP* **05** (2004) 005 [hep-th/0401038]; *Absence of a fuzzy S^4 phase in the dimensionally reduced 5d Yang-Mills-Chern-Simons model*, *JHEP* **07** (2004) 066 [hep-th/0405096]; *Perturbative versus nonperturbative dynamics of the fuzzy

JHEP06(2006)052

$S^2 \times S^2$, *JHEP* **09** (2005) 047 [hep-th/0506205]; *Dynamical aspects of the fuzzy CP^2 in the large N reduced model with a cubic term*, *JHEP* **05** (2006) 061 [hep-th/0405277];
K.N. Anagnostopoulos, T. Azuma, K. Nagao and J. Nishimura, *Impact of supersymmetry on the nonperturbative dynamics of fuzzy spheres*, *JHEP* **09** (2005) 046 [hep-th/0506062].

[44] N. Kawahara and J. Nishimura, *The large N reduction in matrix quantum mechanics: A bridge between BFSS and IKKT*, *JHEP* **09** (2005) 040 [hep-th/0505178].

[45] T. Eguchi and H. Kawai, *Reduction of dynamical degrees of freedom in the large N gauge theory*, *Phys. Rev. Lett.* **48** (1982) 1063.

[46] J. Nishimura and F. Sugino, *Dynamical generation of four-dimensional space-time in the IIB matrix model*, *JHEP* **05** (2002) 001 [hep-th/0111102];
H. Kawai, S. Kawamoto, T. Kuroki, T. Matsuo and S. Shinohara, *Mean field approximation of IIB matrix model and emergence of four dimensional space-time*, *Nucl. Phys.* **B 647** (2002) 153 [hep-th/0204240];
H. Kawai, S. Kawamoto, T. Kuroki and S. Shinohara, *Improved perturbation theory and four-dimensional space-time in IIB matrix model*, *Prog. Theor. Phys.* **109** (2003) 115 [hep-th/0211272];
H. Kaneko, Y. Kitazawa and D. Tomino, *Fuzzy spacetime with* SU(3) *isometry in IIB matrix model*, hep-th/0510263.

[47] H. Aoki, S. Iso, H. Kawai, Y. Kitazawa and T. Tada, *Space-time structures from IIB matrix model*, *Prog. Theor. Phys.* **99** (1998) 713 [hep-th/9802085];
J. Ambjørn, K.N. Anagnostopoulos, W. Bietenholz, T. Hotta and J. Nishimura, *Monte Carlo studies of the IIB matrix model at large N*, *JHEP* **07** (2000) 011 [hep-th/0005147];
J. Nishimura and G. Vernizzi, *Spontaneous breakdown of Lorentz invariance in IIB matrix model*, *JHEP* **04** (2000) 015 [hep-th/0003223]; *Brane world from IIB matrices*, *Phys. Rev. Lett.* **85** (2000) 4664 [hep-th/0007022];
Z. Burda, B. Petersson and J. Tabaczek, *Geometry of reduced supersymmetric 4D Yang-Mills integrals*, *Nucl. Phys.* **B 602** (2001) 399 [hep-lat/0012001];
J. Nishimura, *Exactly solvable matrix models for the dynamical generation of space-time in superstring theory*, *Phys. Rev.* **D 65** (2002) 105012 [hep-th/0108070];
K.N. Anagnostopoulos and J. Nishimura, *New approach to the complex-action problem and its application to a nonperturbative study of superstring theory*, *Phys. Rev.* **D 66** (2002) 106008 [hep-th/0108041];
G. Vernizzi and J.F. Wheater, *Rotational symmetry breaking in multi-matrix models*, *Phys. Rev.* **D 66** (2002) 085024 [hep-th/0206226];
T. Imai, Y. Kitazawa, Y. Takayama and D. Tomino, *Effective actions of matrix models on homogeneous spaces*, *Nucl. Phys.* **B 679** (2004) 143 [hep-th/0307007];
J. Nishimura, *Lattice superstring and noncommutative geometry*, *Nucl. Phys.* **129** *(Proc. Suppl.)* (2004) 121 [hep-lat/0310019];
T. Imai and Y. Takayama, *Stability of fuzzy $S^2 \times S^2$ geometry in IIB matrix model*, *Nucl. Phys.* **B 686** (2004) 248 [hep-th/0312241];
J. Nishimura, T. Okubo and F. Sugino, *Gaussian expansion analysis of a matrix model with the spontaneous breakdown of rotational symmetry*, hep-th/0412194;
S. Bal, M. Hanada, H. Kawai and F. Kubo, *Fuzzy torus in matrix model*, *Nucl. Phys.* **B 727** (2005) 196 [hep-th/0412303];
H. Kaneko, Y. Kitazawa and D. Tomino, *Stability of fuzzy $S^2 \times S^2 \times S^2$ in IIB type matrix models*, *Nucl. Phys.* **B 725** (2005) 93 [hep-th/0506033].

[48] B. Sundborg, *The Hagedorn transition, deconfinement and $\mathcal{N}=4$ SYM theory*, *Nucl. Phys.* **B 573** (2000) 349 [hep-th/9908001].

[49] O. Aharony, J. Marsano, S. Minwalla, K. Papadodimas and M. Van Raamsdonk, *The Hagedorn / deconfinement phase transition in weakly coupled large N gauge theories*, *Adv. Theor. Math. Phys.* **8** (2004) 603 [hep-th/0310285]; *A first order deconfinement transition in large N Yang-Mills theory on a small S^3*, *Phys. Rev.* **D 71** (2005) 125018 [hep-th/0502149].

[50] O. Aharony, J. Marsano, S. Minwalla and T. Wiseman, *Black hole - black string phase transitions in thermal 1+1 dimensional supersymmetric Yang-Mills theory on a circle*, *Class. and Quant. Grav.* **21** (2004) 5169 [hep-th/0406210];
O. Aharony, J. Marsano, S. Minwalla, K. Papadodimas, M. Van Raamsdonk and T. Wiseman, *The phase structure of low dimensional large N gauge theories on tori*, hep-th/0508077.

[51] M. Spradlin and A. Volovich, *A pendant for Pólya: the one-loop partition function of $\mathcal{N}=4$ SYM on $R \times S^3$*, *Nucl. Phys.* **B 711** (2005) 199 [hep-th/0408178].

[52] J.M. Maldacena, *The large N limit of superconformal field theories and supergravity, Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [hep-th/9711200];
S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory, Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109];
E. Witten, *Anti-de Sitter space and holography, Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[53] E. Witten, *Anti-de Sitter space, thermal phase transition, and confinement in gauge theories*, *Adv. Theor. Math. Phys.* **2** (1998) 505 [hep-th/9803131].

[54] S.W. Hawking and D.N. Page, *Thermodynamics of black holes in anti-de Sitter space*, *Commun. Math. Phys.* **87** (1983) 577.

[55] R. Gregory and R. Laflamme, *Black strings and p-branes are unstable*, *Phys. Rev. Lett.* **70** (1993) 2837 [hep-th/9301052].

[56] J.J. Atick and E. Witten, *The Hagedorn transition and the number of degrees of freedom of string theory*, *Nucl. Phys.* **B 310** (1988) 291.

[57] J.T. Yee and P. Yi, *Instantons of M(atrix) theory in pp-wave background*, *JHEP* **02** (2003) 040 [hep-th/0301120].

[58] D.J. Gross and E. Witten, *Possible third-order phase transition in the large-N lattice gauge theory*, *Phys. Rev.* **D 21** (1980) 446.

[59] N. Kim, T. Klose and J. Plefka, *Plane-wave matrix theory from $\mathcal{N}=4$ super Yang-Mills on $R \times S^3$*, *Nucl. Phys.* **B 671** (2003) 359 [hep-th/0306054].

[60] R. Bousso and A.L. Mints, *Holography and entropy bounds in the plane wave matrix model*, hep-th/0512201.

JHEP06(2006)052